Kuwait University	Math. 101	Date:	August 11, 2001
Dept. of Math.&Comp.Sci.	Final Exam.	Duration:	Two hours

Calculators, Mobile Phones and Pagers are not allowed

Answer the following questions: (Each question weighs 4 points)

1. Evaluate the following limit, if it exists $\lim\limits_{x\to\infty}\left(\dfrac{\sqrt{4x^2+1}}{x+2}+x\sin\dfrac{1}{x}\right)$.

2. Classify the discontinuities of f as removable, jump, or infinite where

$$f(x)=\frac{x^2+x}{(x^2-1)\sqrt{x^2}}$$

3. Evaluate: $\displaystyle\int\frac{ds}{\sqrt{s}\cos^2\sqrt{s}}$.

4. Evaluate: $\displaystyle\int_{-1}^{1}\left(t^3+2\sqrt{1-t^2}\right)dt$.

5. Let f be a continuous even function such that $f(x)\geq 0$ for all x in $\mathbb{R}$. If the average value of f on $[0,3]$, $f_{av}=5$, find the area of the region under the graph of f from $x=-3$ to $x=3$.

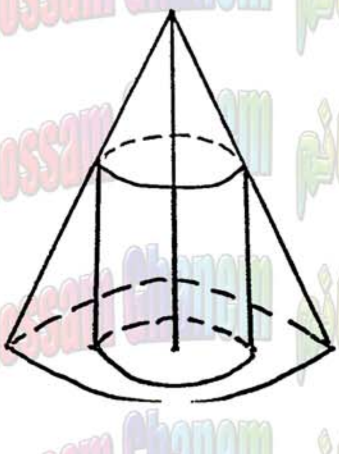
6. Let $f(x)=\displaystyle\int_{x^3+x}^{2}\sqrt{t^2+1}\,dt$. Show that f is a decreasing function and evaluate $f(1)$

7. Find the arc length of the graph of $y=\frac{1}{3}(x^2+2)^{\frac{3}{2}}$ from $x=0$ to $x=1$.

8. Find the area of the region bounded by the graphs of the equations $y=x^3$ and $y=\sqrt{x}$.

9. The region bounded by the graphs of the equations $y=\sqrt{x}$ and $y=x$ is revolved about the line $y=-1$. Find the volume of the resulting solid.

10. Find the dimensions of the hollow cylinder of maximum surface area that can be inscribed in a cone of altitude 10 cm and base radius 5 cm, if the axes of the cylinder and cone coincide.



1. $\lim_{x\to\infty} \left(\frac{\sqrt{4x^2+1}}{x+2} + x\sin\frac{1}{x}\right) = \lim_{x\to\infty}\frac{|x|\sqrt{4+\frac{1}{x^2}}}{x(1+\frac{2}{x})} + \lim_{\frac{1}{x}\to 0^+}\frac{\sin\frac{1}{x}}{\frac{1}{x}} = 2+1 = \boxed{3}$.

2. $f(x) = \frac{x(x+1)}{(x+1)(x-1)\,|x|}$. $\boxed{\lim_{x\to 0^\pm} f(x) = \mp 1}$ ⇒ the graph of f has *jump* discontinuity at $\underline{x=0}$. $\boxed{\lim_{x\to 1^\mp} f(x) = \pm\infty}$ ⇒ the graph of f has *infinite* discontinuity at $\underline{x=1}$. $\boxed{\lim_{x\to -1} f(x) = \frac{1}{2}}$ ⇒ the graph of f has *removable* discontinuity at $\underline{x=-1}$.

3. Put $u = \sqrt{s}$, thus $\frac{ds}{\sqrt{s}} = 2\,du$. $\int \frac{ds}{\sqrt{s}\cos^2\sqrt{s}} = 2\int \sec^2 u\,du = \boxed{2\tan\sqrt{s} + C}$.

4. $\int_{-1}^{1} (t^3 + 2\sqrt{1-t^2})\,dt = 0 + \pi = \boxed{\pi}$.

5. Area under the graph of f from $(x=0)$ to $(x=3) = \int_0^3 f(x)\,dx = 3f_{av} = 15$.

$\boxed{\text{Total area} = 30}$.

6. $f'(x) = -(3x^2+1)\sqrt{(x^3+x)^2+1} < 0$ for all $x \in \mathbb{R}$. ⇒ f is a decreasing function.
$f(1) = \int_2^2 \sqrt{t^2+1}\,dt = 0$.

7. $y' = x(x^2+2)^{\frac{1}{2}} \Rightarrow \sqrt{1+(y')^2} = |x^2+1| \Rightarrow L_0^1 = \int_0^1 (x^2+1)\,dx = \boxed{\frac{4}{3}}$.

8. Points of intersection: $(0,0)$ and $(1,1)$.

Area of rectangle $= (\sqrt{x} - x^3)\,dx \Rightarrow$ Area of the region $= \int_0^1 (\sqrt{x} - x^3)\,dx = \boxed{\frac{5}{12}}$.

9. Volume of washer $= \pi[(\sqrt{x}+1)^2 - (x+1)^2]\,dx \Rightarrow$ Volume of the solid of revolution
$= \int_0^1 \pi[(\sqrt{x}+1)^2 - (x+1)^2]\,dx = \pi[\frac{4x^{\frac{3}{2}}}{3} - \frac{x^3}{3} - \frac{x^2}{2}]_0^1 = \boxed{\frac{\pi}{2}}$.

10. Let S be the surface area of the cylinder of radius r and altitude h. Since, $\frac{r}{5} = \frac{10-h}{10}$, then $S = 2\pi rh = 4\pi(5r - r^2)$, where $0 \le r \le 5$. $\frac{dS}{dr} = 4\pi(5-2r)$, $\frac{d^2S}{dr^2} = -8\pi$. The only critical number for S is $r = \frac{5}{2}$ and $\left.\frac{d^2S}{dr^2}\right|_{r=\frac{5}{2}} < 0$. Thus S is maximum at $r = \frac{5}{2}$ cm and $h = 5$ cm and $S_{\max} = 25\pi$ cm^2.

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